Estimations of angles between some axes in radio pulsars from catalog at 1000 MHz

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There are some known theoretical models used for investigations of pulsars at present. One of the most important parameters for the check of existing pulsar models is the angle β between an axis of an emission cone (a direction of a magnetic moment vector) and a pulsar rotation axis. Some results of calculations of this angle are presented. For such calculations the sample of pulsars from the paper of Van Ommen, Alessandro et al. (1997) was used. The estimations of the angle β was carried out in three ways. The first two ways are based on some statistical relationships. The third way uses observable values of positional angles and shapes of average profiles for individual pulsars only. The distributions of angles β obtained by three methods and some results of their comparison are discussed.

Introduction

There are some known theoretical models used for investigations of pulsars at present. The obtained width of a pulsar pulse profile in the hollow cone model [1] is determined by the superposition of the following factors (Fig. 1): μ — the direction of the magnetic moment vector (the axis of an emission cone), Ω — the axis of a pulsar rotation, L — the line of sight of the observer, ζ — the angle between L and Ω, β — the angle between μ and Ω, θ — the angular radius of an emission cone.

One of the most important parameters for the check of the known pulsar models is the angle β. The angle β allows:

- to understand features of a pulsar radiation;
- to make a conclusion on ways of the pulsar evolution;
- to find out the reason of the interpulse radiation;
- to make a conclusion on the applicability of the assumption about a dipole magnetic field in pulsar magnetospheres;
- to choose an adequate model of a pulsar.

The estimations of the angle β was carried out in three ways. The first two ways are based on some statistical relationships. The third way uses observable values of position angles and shapes of average profiles for individual pulsars. Some results of calculations of this angle are presented.

The method of calculations

For calculations of β the sample of pulsars from [2] was used. We considered 80 pulsars with known pulse profiles for the first way of calculations, and 43 pulsars with monotonous behavior of a position angle for the second way. In Fig. 2 the dependence of the observable pulse width W_{10} (at the 10% level) on the pulsar period is presented in logarithmical scale for our sample.

The relation between parameters under consideration is set by the equation:

$$\cos\theta = \cos\beta \cdot \cos\zeta + \sin\beta \cdot \sin\zeta \cdot \cos\frac{W_{10}}{2}. \qquad (1)$$

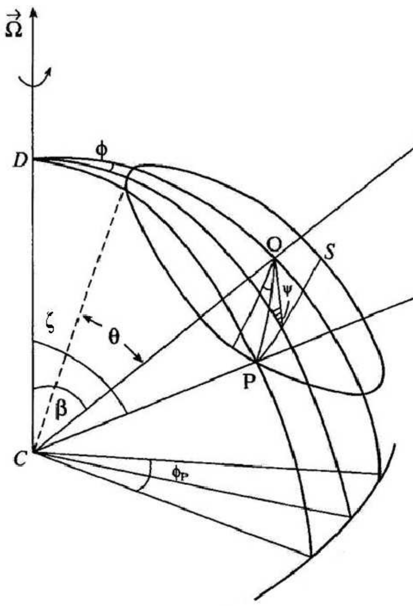

Figure 1: The hollow cone model.

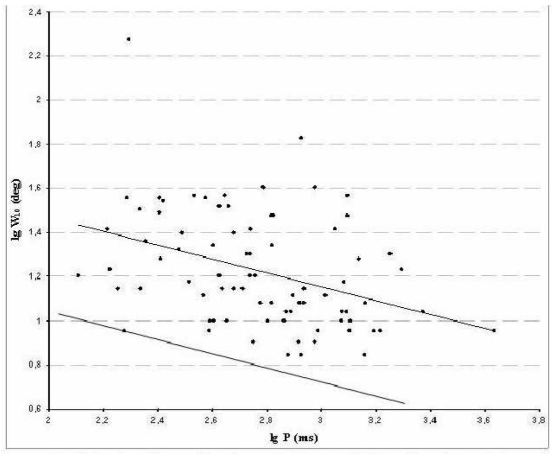

Figure 2: The dependence of the observable pulse width W_{10} on the pulsar period.

In the case, when the line of sight passes through the center of the emission cone, $\beta = \zeta$. Taking into account the lower β limit for the sample of considered pulsars (Fig. 2), one can derive:

$$\theta = 2.63^\circ \cdot P^{-0.32}. \tag{2}$$

Using the relationships (1) and (2) one can obtain:

$$\beta = \arcsin \frac{\sin\left(\frac{\theta}{2}\right)}{\sin\left(\frac{W_{10}}{4}\right)}. \tag{3}$$

The way of the angle β calculations using the maximum derivative of the position angle of polarization consists in the following. We suggest that generation of radio waves occurs in that area where there are no effects of propagation (distortions, direction changes) and hence the value of the position angle has been not deformed.

It is necessary to take into account the widening effect (owing to an approach to a rotation pole) and the width decrease for the line of sight passing not through the center of the emission cone. It is not clear in advance the contribution of which effect is higher, thus we suggest them to be equal. Then we can define the dependence $W_{10}(P)$ by a straight line fitted in the sample data. The line $\theta = 6.76^\circ \cdot P^{-0.32}$ was obtained with the least squares method.

Dependence of the position angle ψ of polarization plane on the longitude φ is expressed by the following formula:

$$\tan\psi = \frac{\sin\beta \cdot \sin\varphi}{\sin\zeta \cdot \cos\beta - \cos\zeta \cdot \sin\beta \cdot \cos\varphi}. \tag{4}$$

Maximum derivative of the position angle can be determined from the equality:

$$\left(\frac{d\psi}{d\varphi}\right)_{max} = \frac{\sin\beta}{\sin(\zeta - \beta)}. \tag{5}$$

Thus having two equations for the angles β and ζ, and denying the assumption that $\beta = \zeta$ one can obtain more accurate values of these angles. Values of derivatives were calculated for the data received at different, but almost equal frequencies.

Using the following denotations:

$$\cos\theta = B,\ \left(\frac{d\psi}{d\varphi}\right)_{max} = C,\ \cos\left(\frac{W_{10}}{2}\right) = D, \qquad (6)$$

and transforming the initial system of equations, one can obtain

$$\begin{array}{l} C^2(1-D^2)y^4 + 2C(1-D)y^3 + [1+2DC^2(1-D)]y^2 + 2C(D-B^2)y + D^2C^2 - B^2(1+C^2) = 0, \\ \tan\beta = \frac{C\sqrt{1-y^2}}{1+Cy}, \qquad y = \cos\zeta. \end{array} \qquad (7)$$

However, it is impossible to define a sign of the maximum derivative by using observations of the main pulse only. Therefore it is necessary to solve the equations for $C > 0$ and $C < 0$ separately. It is worth noting, that it is not possible to obtain the solution of the considered equations for some pulsars, using the observed values D, C, B. It means that in a number of pulsars this model does not work.

There is a third way to calculate the angles β. This way uses observable values of position angles and shapes of average profiles for individual pulsars. In this case, original equations form the complete system for calculations of the values of the angles θ, ζ and β [3]:

$$\left\{\begin{array}{rcl} \sin\beta & = & C\sin(\zeta-\beta), \\ \cos\theta & = & \cos\zeta\cdot\cos\beta + D\sin\beta\sin\zeta, \\ \theta & = & n(\zeta-\beta) \end{array}\right. \qquad (8)$$

As the observed pulsar profiles have various forms, a coefficient n has a different value depending on an intensity in the center of the observed profile [3].

E. g., for $n = 4$ the solution for $y = \cos\zeta$ could be obtained from the equation:

$$(CD+y+Cy^2(1-D))\sqrt{(C^2+2Cy+1)^3} - 8y^4 - 16Cy^3 - 4(3C^2-2)y^2 - 4C(C^2-3)y - C^4 + 6C^2 - 1 = 0; \quad (9)$$

for $n = 2$:

$$\begin{array}{l} 2C^3(1-D)^2y^5 + \left[C^4(1-D)^2 + C^2(D^2-6D+5) - 4\right]y^4 + \\ \quad + 2C\left[C^2(1+D-2D^2) - 2 - D\right]y^3 + \left[2DC^4(1-D) - C^2(2D^2-6D+7) + 5\right]y^2 + \\ \quad\quad + 2C\left[C^2D^2 + D(1+C^2) - 2(C^2-1)\right]y + C^2D^2(1+C^2) - (C^2-1)^2 = 0; \end{array} \quad (10)$$

for $n = 3/2$:

$$\left[2(y+C) - \sqrt{C^2+2Cy+1}\right]\sqrt{\frac{1+\frac{C+y}{\sqrt{C^2+2Cy+1}}}{2}} - Cy^2(1-D) - y - CD = 0; \qquad (11)$$

and for $n = 5/4$:

$$\left(1 + \frac{2(C+y)}{\sqrt{C^2+2Cy+1}} - \sqrt{2\left(1+\frac{C+y}{\sqrt{C^2+2Cy+1}}\right)}\right)\times$$
$$\times\sqrt{\frac{1+\sqrt{\frac{1+\frac{C+y}{\sqrt{C^2+2Cy+1}}}{2}}}{2}} - \frac{Cy^2(1-D)+y+CD}{\sqrt{C^2+2Cy+1}} = 0. \qquad (12)$$

The coefficient n may have other values as well. The angles β (β_3) obtained using of the equations (9)-(12) are systematically higher than β_1 and β_2. Detailed discussion of these results will be presented in separate publications.

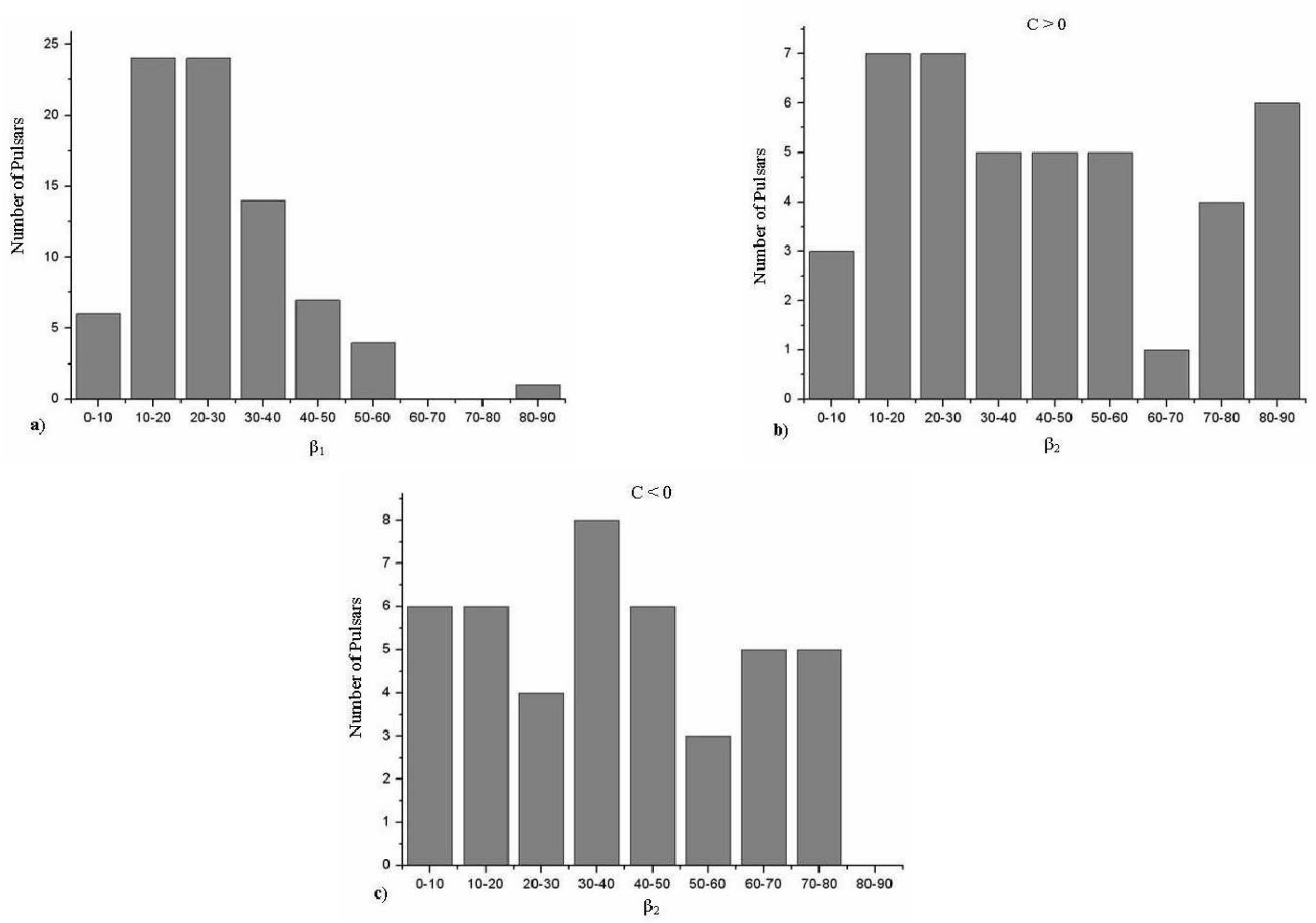

Figure 3: The distributions of pulsar numbers on values β: a) β_1; b) β_2 for $C > 0$; c) β_2 for $C < 0$.

Results and conclusions

The angles calculated in the first way are denoted with β_1. For the obtained angles the distribution of pulsar numbers on values β_1 is plotted (Fig. 3a). It is easily to see from this histogram, that the majority of pulsars have values of β_1 from 10° to 30° degrees with the mean value $\langle\beta_1\rangle = 25.87^\circ$.

The angles calculated in the second way are denoted with β_2. For the obtained angles the distribution of pulsar numbers on values β_2 is plotted. The mean values are $\langle\beta_2\rangle = 43.22^\circ$ for $C > 0$ (Fig. 3b) and $\langle\beta_2\rangle = 38.35^\circ$ for $C < 0$ (Fig. 3c). The obtained values of β_1 are systematically less than the values of β_2. This was expected, because the values of β_1 are the lower estimations of the real tilt angle.

References

[1] Manchester R., Taylor J. "Radio Pulsars". "Mir", Moscow (1980) (in Russian)

[2] van Ommen T. D., D'Alessandro F., Hamilton P. A., McCulloch P. M. Mon. Notic. Roy. Astron. Soc., V. 287, pp. 307-327 (1997)

[3] Malov I. F. "Radio Pulsars". Nauka, Moscow (2004) (in Russian)